BRIEF COMMUNICATIONS

THE ANALYTIC REPRESENTATION OF THE PULSE ENERGY IN THE WAKE

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From the equation of balance for the pulse energy we find the distribution of the average turbulent energy in the automodel region of a two-dimensional wake.

The equations of balance for the momentum and the kinetic energy of turbulence in a remote part of a two-dimensional wake in an incompressible fluid can be written as

$$U_1 \frac{\partial U}{\partial x_1} = -\frac{\partial}{\partial x_2} \left(\overline{v_2' v_1'} \right),$$

$$U_1 \frac{\partial E}{\partial x_1} = -\frac{\partial}{\partial x_2} \left[\overline{v_2' \frac{1}{2} \sum_{i=1}^{3} v_i'^2} + \frac{1}{\rho} \overline{v_2' p'} \right] - \overline{v_2' v_1'} \frac{\partial U}{\partial x_2} - \nu \sum_{i,\alpha=1}^{3} \overline{\left(\frac{\partial v_i'}{\partial x_\alpha} \right)^2}, \tag{1}$$

where $E = \sum_{i=1}^{3} \overline{v_i'^2}/2$; the bar denotes the time average.

We can express the turbulent friction in Prandtl's form:

$$\overline{v_2' v_1'} = -\nu_\text{т} \frac{\partial U}{\partial x_2}, \quad \nu_\text{т} = (U_1 - U_0)\,\delta \tag{2}$$

and put, for example, as in [1]

$$\overline{v_2' \frac{1}{2} \sum_{i=1}^{3} v_i'^2} + \frac{1}{\rho} \overline{v_2' p'} = -\nu_\text{т} \frac{\partial E}{\partial x_2}, \quad \nu \sum_{i,\alpha=1}^{3} \overline{\left(\frac{\partial v_i'}{\partial x_\alpha} \right)^2} = c\nu_\text{т} \frac{E}{\delta^2}, \tag{3}$$

where c is a constant to be determined experimentally. Introducing the new variables

$$u = U_1 - U, \quad \frac{u}{u_0} = f(\eta), \quad \eta = \frac{x_2}{\delta},$$

$$\frac{u_0}{U_1} = \psi(x_1), \quad \frac{E}{E_0} = h(\eta), \quad \frac{E_0}{U_1^2} = \varphi(x_1)$$

and using the integral condition in the form

$$\psi\delta = \frac{c_x d}{4} \frac{1}{\int_0^\infty f d\eta} = \text{const}, \tag{4}$$

we find that there is similarity if

$$\delta'/\psi = -\psi'\delta/\psi^2 = -\varphi'\delta/2\varphi\psi = B = \text{const}, \tag{5}$$
$$\psi^2/\varphi = D = \text{const}.$$

Then (1) takes the form

$$f'' + zf' + f = 0, \tag{6}$$

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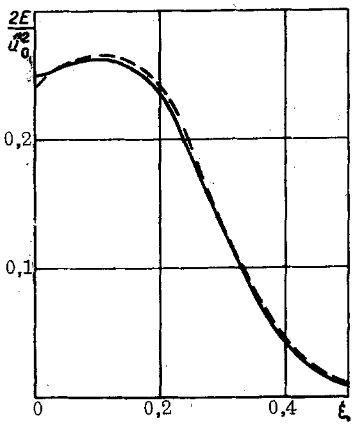

Fig. 1. Experimental data from [2] (continuous curve) and computation (dotted curve).

$$h'' + zh' + (2 - c/B)\,h = -Df'^2, \tag{7}$$

where the primes denote differentiation with respect to $z = \sqrt{B}\eta$.

We know the solution of Eq. (6) with the following boundary conditions:

$$f = 1 \text{ for } z = 0,\ f \to 0 \text{ as } z \to \infty$$

to be

$$f = \exp(-z^2/2). \tag{8}$$

The pulse-energy profile must satisfy the boundary conditions

$$h = 1 \text{ for } z = 0,\ h \to 0 \text{ as } z \to \infty, \tag{9}$$

and also the symmetry condition

$$h' = 0 \text{ for } z = 0. \tag{10}$$

We can show that we can describe the experimental results completely satisfactorily by taking $c = 3B$. Then the solution of Eq. (7), taking account of (8), with the boundary conditions (9) can be written as

$$h = z\{(3/2)D\sqrt{\pi}[1 - \Phi(z\sqrt{2})] - (1 + 2D)\sqrt{\pi/2}\,[1 - \Phi(z)]\} + (1 + 2D)\exp(-z^2/2) - 2D\exp(-z^2), \tag{11}$$

where

$$\Phi(x) \equiv \sqrt{2/\pi}\int_0^x \exp(-t^2/2)\,dt.$$

It follows from this that

$$h'(0) = (1 + 2D)\sqrt{\pi/2} - 3D\sqrt{\pi}/2.$$

Using (10), we have

$$D = \frac{\sqrt{2}}{3 - 2\sqrt{2}} \approx 8.2.$$

In Fig. 1 there is a comparison between the computed and the experimental values [2]. The ordinate is, from (5), (11):

$$\frac{2E}{u_0^2} = \frac{2}{D}h = 3\sqrt{\pi}\,z\,[\Phi(z) - \Phi(z\sqrt{2})] + (2/D + 4)\exp(-z^2/2) - 4\exp(-z^2). \tag{12}$$

The abscissa is $\xi = x_2/\sqrt{dx_1}$, and we can find that $z = 5.42\xi$, since, by (4) and (5),

$$\delta/\sqrt{B} = \sqrt{c_x\sqrt{B}\,dx_1/2}$$

and, by experiment [2], $\sqrt{B} = 1/R_T = 0.08$, $c_x = 0.85$.

Using (8) and (12) it is easy to compute the ratio of the total intensities of the change in the average velocity and pulse velocity in the automodel region:

$$\frac{\int_{-\infty}^{\infty} u^2\,dx_2}{\int_{-\infty}^{\infty} 2E\,dx_2} = \frac{D}{2}\,\frac{\int_0^{\infty} f^2 d\eta}{\int_0^{\infty} h\,d\eta} = 2,$$

which agrees exactly with experiment (cf. [3], p. 173, Fig. 7, 1b).

NOTATION

U is the longitudinal component of average velocity;
U_1 is the velocity of the undisturbed flow;
v_i', p' are the velocity and pressure pulsations;

ν is the kinematic viscosity coefficient;
δ is the relative wake width;
ρ is the density;
d is the height of middle section of body;
c_x is the drag coefficient.

Subscript

0 denotes the wave axis.

LITERATURE CITED

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3. A. A. Townsend, The Structure of Turbulent Flow with Transverse Shear [Russian translation], IL (1959).